Sub code: 0081

NEB-XII
Model Question
Mathematics
2079/2023

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Time: 3hrs **Full Marks:** 75

Attempt all questions.

Group "A" **11×1 = 11**

1) What is an arrangement of the n natural numbers called?
   A) Induction B) Permutation C) Combination D) Expectation

2) Let 1, w, w^2 be the cube roots of unity. Under which operation is the set A= {1, w, w^2} closed?
   A) Addition B) Subtraction C) Multiplication D) Division

3) What is the domain of $\sin^{-1}x$?
   A) $x \geq 1$ or $x \leq -1$ B) $(-\infty, \infty)$ C) $-1<x<1$ D) $-1 \leq x \leq 1$

4) ABCD is a parallelogram.Which one of the following represents area of the parallelogram?
   A) Magnitude of vector product of two vectors along AB and BD.
   B) Magnitude of vector product of two vectors along AB and DC.
   C) Magnitude of vector product of two vectors along AC and BC.
   D) Magnitude of vector product of two vectors along AB and AD.

5) If a conic section haseccentricity(e) = $\frac{\sqrt{a^2-b^2}}{a}$, what is the equation of that conic section?
   A) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ B) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ C) $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$ D) $\frac{x^2}{b^2} + \frac{y^2}{b^2} = 1$

6) If $\cos\Theta = \frac{-1}{2}$ for integer (n), what is the general value of Θ ?
   A) $2n\pi \pm \frac{2\pi}{3}$ B) $n\pi + (-1)^n \frac{\pi}{3}$ C) $n\pi - \frac{\pi}{3}$ D) $n\pi + \frac{\pi}{3}$

7) Let A and B be two dependent events. If P (A) = $\frac{1}{2}$, P (B) = $\frac{3}{4}$ and P (A ∩ B) = $\frac{2}{5}$, what is the value of P(A/B) ?
   A) equal to P(B/A) B) equal to P(A) C) less than P(A ∩ B) D) less than P(B/A)

8) The edge of a cube increases from 10 cm to 10.025 cm. What would be the approximate increment in volume?
   A) 10^3 cm^3 B) 10.025^3 cm^3 C) 7.5187 cm^3 D) 7.5 cm^3

9) What is the integrating factor of the differential equation $\cos^2 x \frac{dy}{dx} + y = 1$?
   A) tan x B) $e^{\tan x}$ C) $\sec^2 x$ D) $e^{\sec 1x}$

10) What is the number of solutions of the system of linear equations x + y = 5 and x + y = 7 ?
   A) One solution B) No solution
   C) Infinite solutions D) More than one solution

11) Forces P and Q are acting along ceiling and floor of a rectangular room. What is the nature of the forces?
   A) Like B) unlike C) collinear D) parallel

   OR

   If $\Delta y_t = y_{t+1} - y_t$, then $\Delta^2 y_t$ is equal to…
   A) $y_{t+2} - y_{t+1}$ B) $y_{t+1} - y_t$ C) $y_{t+2} - y_{t+1} + y_t$ D) $y_{t+2} - 2y_{t+1} + y_t$

Group "B" 8×5 = 40

12) For any positive integer n, $(a+x)^n = c_0 a^n + c_1 a^{n-1}x + c_2 a^{n-2}x^2 + \ldots\ldots + c_n x^n$

a) How many terms are there in the expressions? [1]

b) Write the binomial coefficients in the expansion. [1]

c) Write the general term of the expansion. [1]

d) Write the relation among C(n,r-1), C(n+1,r) and C(n,r). [1]

e) What is the value of $C_0 + C_1 + C_2 + \ldots + C_n$? [1]

13) a) Using the principle of mathematical induction, showthat: $1+2+3+\ldots. +n < \frac{1}{8}(2n+1)^2$ [3]

b) Find the quadratic equation whose one of the roots is $2+\sqrt{3}$. [2]

14) a) Given $y = \sin^{-1}x$ and $y>0$, express cosy and tany in terms of x. [3]

b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ for $\vec{a} \neq (0,0)$., show that $\vec{b} = \vec{c}$. [2]

15) The price in Rupees (X) and demand in unit (Y) of 6 days of a week is given as: [5]

X	10	12	13	12	16	15
Y	40	38	43	45	37	43

Calculate the Pearson's coefficient of correlation and the regression coefficients of X on Y.

16) a) Define L.Hospital's rule. [1]

b) Write the slope of the tangent and normal to the curve y = f(x) at (x_1,y_1). [1+1]

c) Write the integral of $\int \frac{1}{x^2+a^2}dx$ [1]

d) What is the integral of $\int Sinhx\, dx$ [1]

17) a) Solve: $\frac{dy}{dx} = \frac{y}{x}$ [2]

b) Verify the Rolle's theorem for $f(x)=x^2+3x-4$ in [-4,1]. [3]

18) Using simplex method, maximize P(x,y)=15x+10y subject to $2x+y \leq 10$, $x+3y \leq 10$, $x, y \geq 0$ [5]

19) A particle is projected with a velocity 'v' and greatest height is 'H', prove the horizontal range R is:

$R=\sqrt{H(\frac{v^2}{2g} - H)}$

OR,

The cost function C (x) in thousands of rupees for producing x units of maths textbooks is given by

$C(x)=30+20x-0.5x^2$, $0 \leq x \leq 15$

a) Find the marginal cost function. [3]

b) Find the marginal cost for producing 12,000 maths textbooks. [2]

Group "C" **3×8 = 24**

20) a) Using matrix methods, solve the following system of linear equations :
x+y+z = 4, 2x+y-3z = -9,2x-y+z = -1 [4]

b) Apply De-moivre's theorem to find the value of $\left[2(cos15^\circ + isin15^\circ)\right]^6$ [2]

c) Prove that $\left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right)\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots\right) = 1$ [2]

21) a) Find the direction cosines of the line joining the points (4,4,-10) and (-2,2,4). [2]

b) Find the angle between the two diagonals of a cube. [4]

c) Find the vertices of the conic section:
$16(y-1)^2 - 9(x-5)^2 = 144$. [2]

22) a) If the limiting value of $\frac{(f(x)-5)}{x-3}$ at x= 3 is 2 by using L' Hospital' rule, find the appropriate value of f(x). [2]

b) Write any one homogeneous differential equation in (x,y) and solve it. [3]

c) The concept of anti-derivative is necessary for solving a differential equation. Justify this statement with example. [3]

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